$$CD = \frac{ab^2}{b^2+c^2},\ AE = \frac{bc^2}{a^2+c^2},\ BF = \frac{a^2c}{a^2+b^2};$$

$$\therefore\ \alpha = \frac{b^2}{b^2+c^2},\ \beta = \frac{c^2}{a^2+c^2},\ \gamma = \frac{a^2}{a^2+b^2};$$

$$\therefore\ \frac{\Delta'}{\Delta} = \frac{2a^2b^2c^2}{(a^2+b^2)(a^2+c^2)(b^2+c^2)}.$$

Compare this result with that in 2.

5. *Let the points D, E, F, be the feet of the perpendiculars let fall from the centre of the inscribed circle.*

Denote the radius of the inscribed circle by ρ and put $\frac{1}{2}(a+b+c) = s$; then $\alpha = (\rho \div a)\cot\frac{1}{2}C$, $\beta = (\rho \div b)\cot\frac{1}{2}A$, $\gamma = (\rho \div c)\cot\frac{1}{2}B$. Therefore

$$\frac{\Delta'}{\Delta} = \frac{2\rho^3}{abc}\cot\tfrac{1}{2}A\cot\tfrac{1}{2}B\cot\tfrac{1}{2}C = \frac{\rho^2 s}{abc} = \frac{2(s-a)(s-b)(s-c)}{abc}$$

$$= \frac{(a+b-c)(a+c-b)(b+c-a)}{4abc}.$$

Compare this result with that in 3.

FIVE GEOMETRICAL PROPOSITIONS.

BY PROF. ELIAS SCHNEIDER, MILTON, PA.

I. LET A, B, C, D, &c., be the angular points of a regular polygon of n sides, and let AB, one of the equal sides, aqual unity; then will AB be contained once in AC, the chord which contains *two* of the equal sides, with a remainder which call x. Then is $\sqrt{(1-x)}$ = one side of a polygon of $2n$ sides inscribed in a circle whose radius is *one.*

II. AB will be contained twice in AD, the chord which contains *three* of the equal sides, with a remainder which call y. Then is $\sqrt{(1-y)}$ = one side of a polygon of n sides inscribed in a circle whose radius is *one.*

III. If the polygon be a Nonagon, AB will be contained twice in AE, the chord which contains *four* of the equal sides, with a remainder which call x. Then is $\sqrt{(1-x)}$ = one side of a polygon of 18 sides inscribed in a circle whose radius is *one.*

IV. If in Prop. II the polygon be also a Nonagon, then is

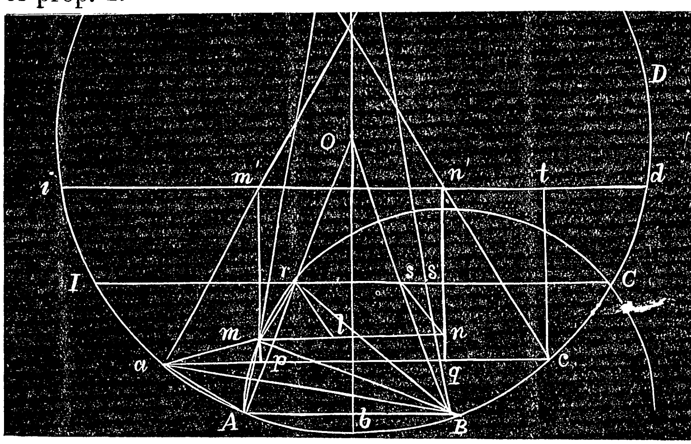
$$\sqrt{(1-x)} = x - y.$$

V. If the polygon be a Decagon, AB will be contained twice in AD, the chord which contains three of the equal sides, with a remainder which call z. Then is $\sqrt{(1-z)} = z$ = one side of a decagon inscribed in a circle whose radius is *one.*

[As the foregoing propositions are new, or at least original with Prof. Schneider (see ANALYST, Vol. I, p. 37), and have never been demonstrated geometrically, so far as we know; and as Prof. S. declined to submit his *proof*, we have attempted their demonstration, and submit the following sketch of our method and result.—Ed.]

It will be seen that the Figure is drawn for a Nonagon and that ac, IC, and id are chords containing 2, 3 and 4 of the equal sides, respectively.

The $\angle AOB = ABr = 2AFB = 2ABm$. Draw mn parallel, and mp and nq perpendicular to ac. Then, because aB is perpendicular to Am, we may easily prove that $am = aA$, and $ap = qc = \frac{1}{2}AB = \frac{1}{2}$; therefore $mn = pq = x$, of prop. I.



By Euclid, Prop. D, B'k VI, we have $Am \times Bn + AB \times mn = An \times Bm$; or, because $Bn = Am$, $Am^2 + x = 1$. Transpos'g and extracting root we have $Am = \surd(1 - x)$, which proves Prop. I. And in like manner, substituting rs $(= y)$ for mn, we get $Ar = \surd(1 - y)$, which proves Prop. II. The proof in both these cases is obviously independent of any particular value of n.

If the polygon is a Nonagon, ch will intersect id at an angle of 60° and therefore cdn' is an equilateral triangle and ct is equal and parallel to qn; therefore $td = n't = qc = \frac{1}{2}$; therefore $n'd = 1$, and $m'n' = mn = x$, and the demonstration of I applies also to III.

Join sn and draw rl parallel to sn, then, if the polygon is a Nonagon, the triangle lmr will be equilateral and $mr = \surd(1 - x) = ml = mn - rs = x - y$, which proves Prop. IV.

If the polygon is a Decagon, we may easily prove that the triangle Ars is isosceles; therefore $Ar = rs$. But by Prop. II, $Ar = \surd(1 - rs) = \surd(1 - Ar)$. Or putting z for Ar, $z = \surd(1 - z) =$ one side of a Decagon inscribed in a circle whose radius is *one*.

COR. If we put $R = AB$, the functions $\surd(1 - x)$, $\surd(1 - y)$ and $\surd(1 - z)$, in the above cases, become $\surd(R^2 - Rx)$, $\surd(R^2 - Ry)$ and $\surd(R^2 - Rz)$.